Econ 424 Project Summer 2024

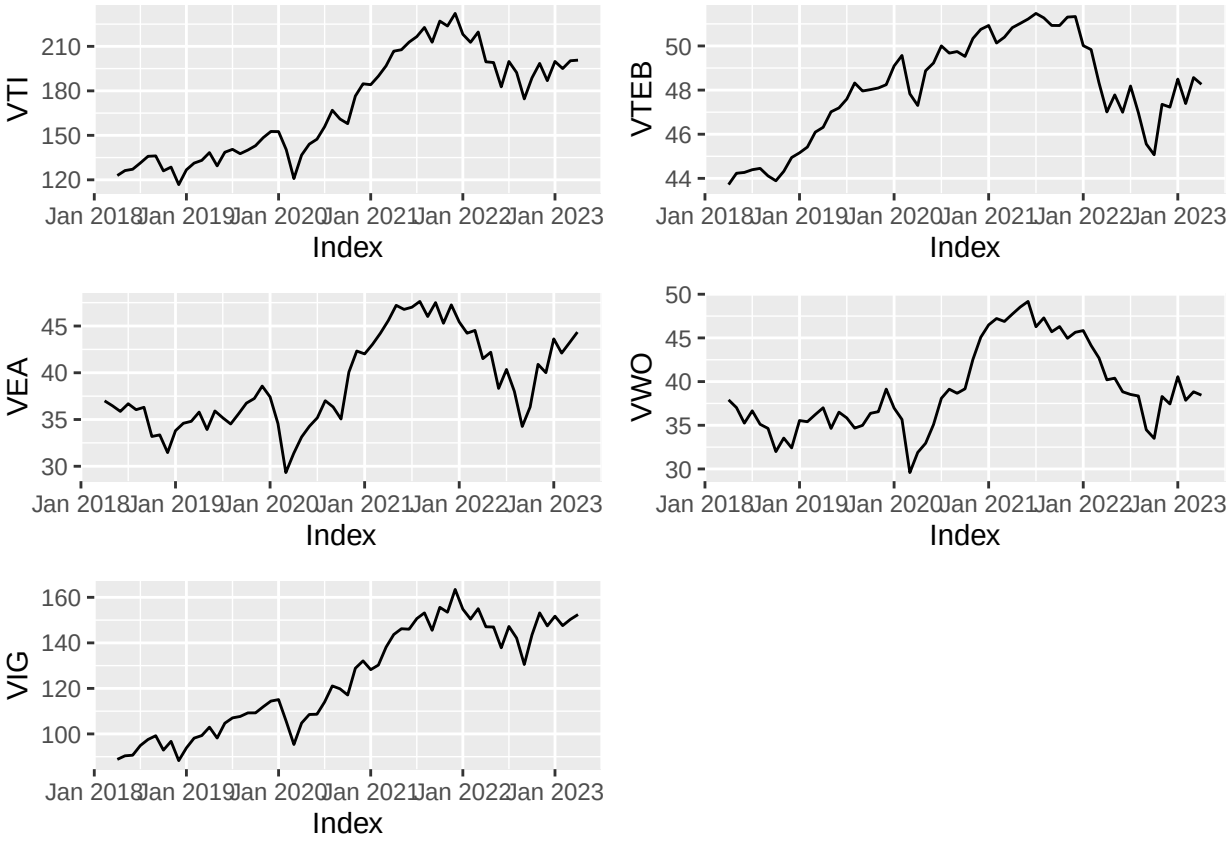
Analysis

Summary of ETFs

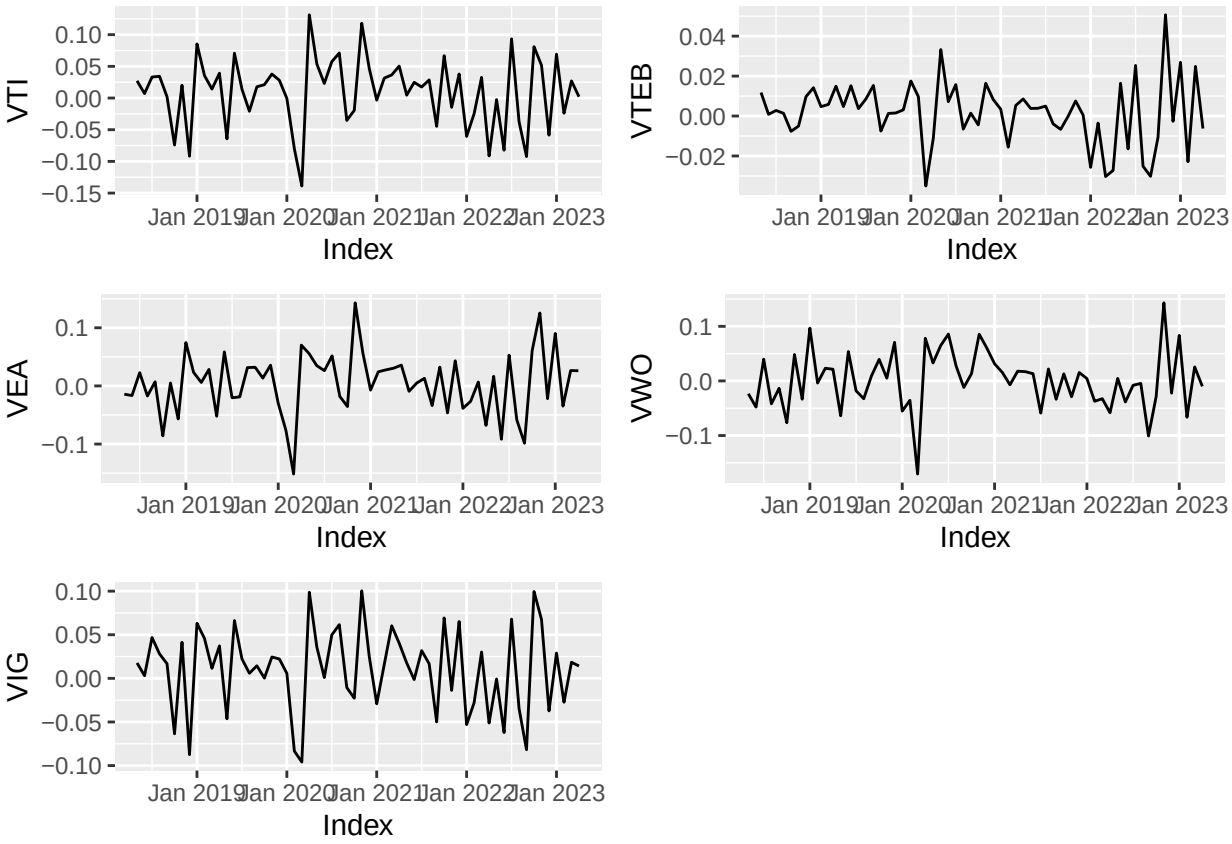
1. US Stocks. Vanguard Total Stock Market ETF (VTI): This ETF holds stocks that represent the investable US stock market, including a diverse range of stock sizes. Overall, this fund is made to represent the characteristics of the US stock market.
2. Municipal Bonds. Vanguard Tax-Exempt Bond Index ETF (VTEB): This ETF holds stocks from US governmental agencies, specifically those that are exempt from federal tax.
3. Foreign Developed Stocks. Vanguard FTSE Developed Markets ETF (VEA): This ETF focuses on stock from companies outside the US, but focuses on developed economies like Europe, Canada, Japan, and the Pacific region, excluding emerging markets.
4. Emerging market stocks. Vanguard FTSE Emerging Markets ETF (VWO): This ETF includes stocks from emerging markets, aiming to track the performance and characteristics of the All Cap China A Inclusion Index. It includes countries like China, India, Taiwan, and other emerging economies.
5. Dividend Growth Stocks. Vanguard Dividend Appreciation ETF (VIG): This ETF includes stocks within the U.S. Equity market with annually increasing dividend payouts. It is mainly focused on consumer stocks, and only selects companies that have increased payouts for 10 consecutive years. It includes holdings in Apple, Microsoft, Broadcom, and other large US-based corporations.

Return Calculation and Sample Statistics

Monthly Prices and Simple Returns

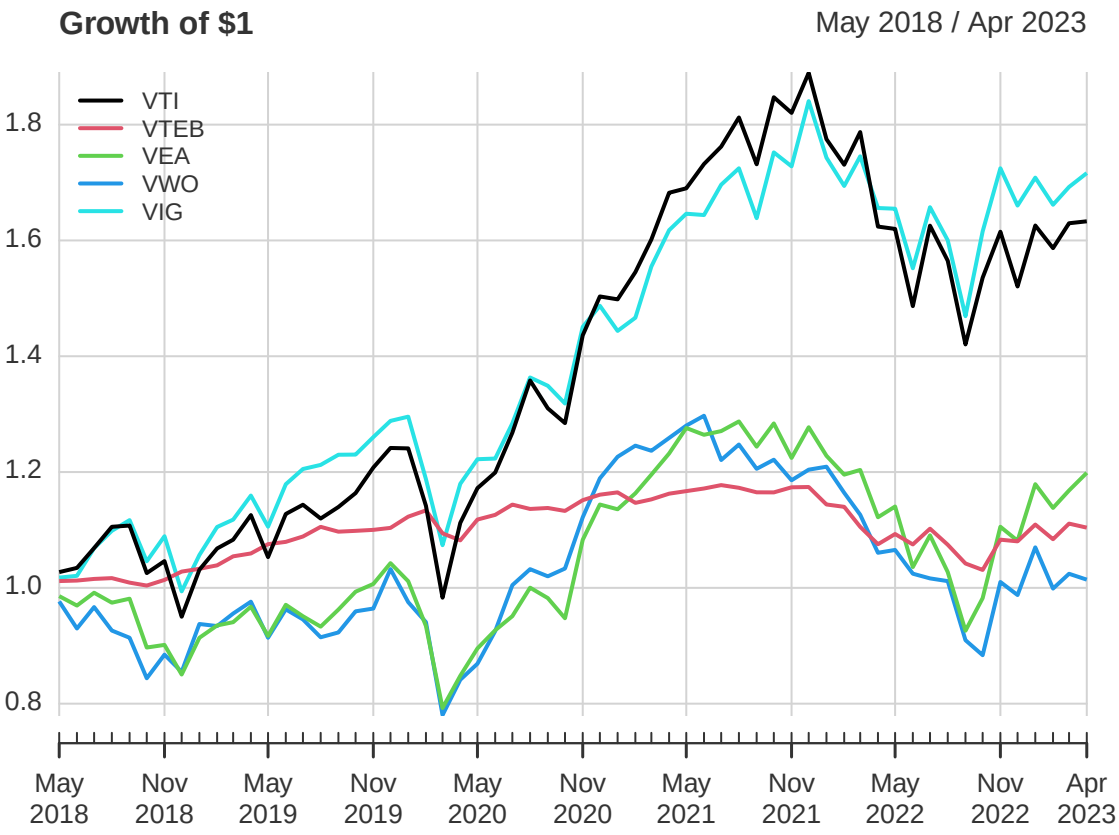


On average, all the ETFs have prices that seem to move together. From the graphs, we can see that all prices dipped during late 2018 and early 2022, and all ETFs had rising prices from early 2020 to early 2022, following by a decrease. VTI and VIG have similar prices, going above 100, and VEA, VTEB, and VWO all have prices ranging between 30 and 50 dollars. We can also see that VTI and VIG suffered the least decrease in prices in late 2022, while VEA, VWO, and VTEB depict large price decreases in this time. World events that could explain these changes would be the trade war between the US' Trump Administration and China in late 2018, the economic impact of COVID-19 in early 2020, and the Russia-Ukraine war in late 2022. Some degree of the price drop in late 2022 could also be explained by the sharp increase in interest rates in late 2022 by the Fed, which caused the Silicon Valley bank to collapse.



On the return graph, we can see that all 5 ETFs react similarly to large economic downturns such as that in the beginning of early 2020 during COVID-19. The return plots of each graph show sharp downward spike at this time, followed by an immediate increase back to the same return or slightly higher. Out of the 5, VIG and VTI display the highest variance, as we can see from the large amplitudes. None of these returns appear to be covariance stationary, as rolling means and variance wouldn't be constant over time due to inequal magnitudes of consecutive upward and downward spikes.

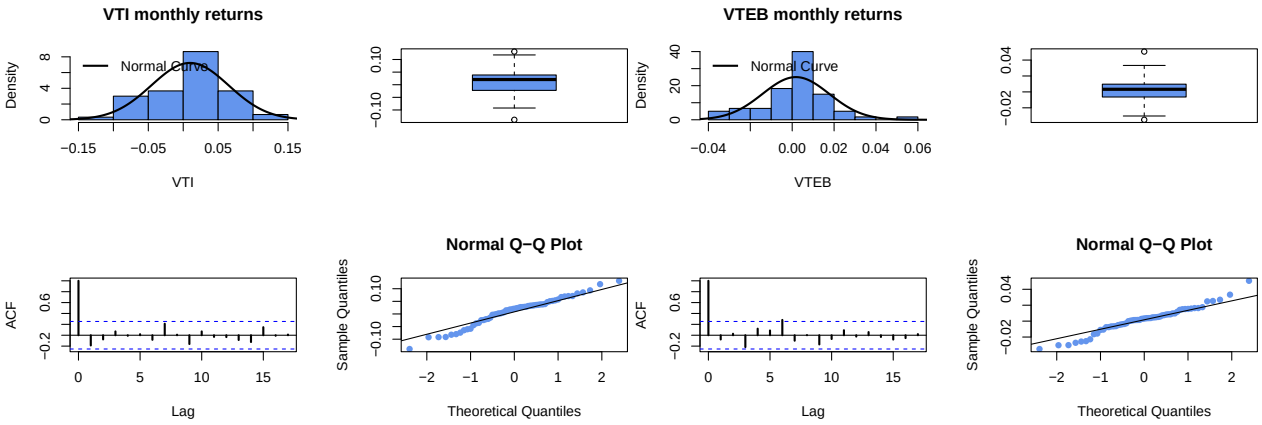
Equity curve

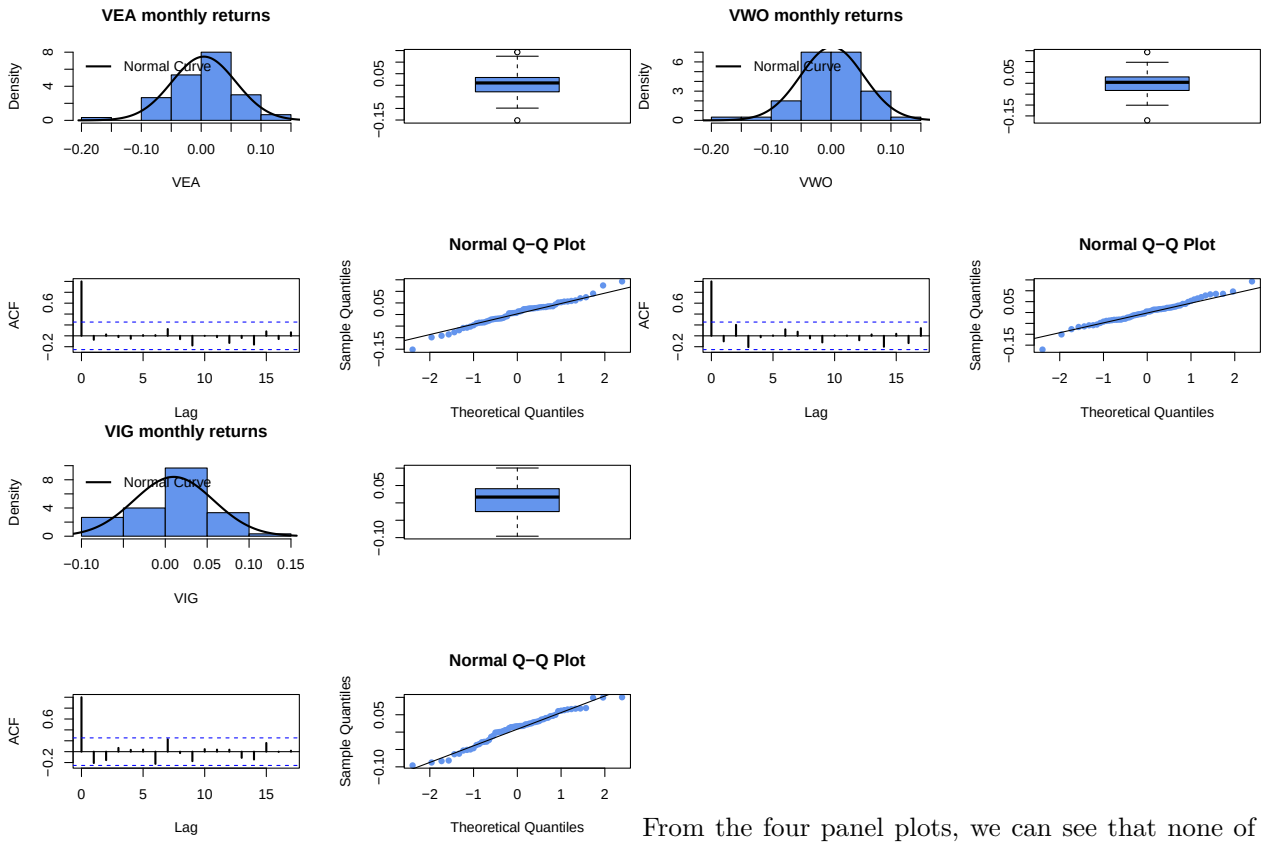


Out of the 5 ETFs, VTI and VIG show the highest equity growth rate. This aligns well with our expectations of their growth rates based on the ETF descriptions. VTI and VTEB both represent specifically the US stock market - the dominant global stock market, with VTI focusing on embodying the market's general characteristics, while VTEB focuses on highlighting stocks with high equity and growth rates. VTEB shows the least overall change, likely due to this ETF mainly consisting of government stocks. VEA and VWO show more fluctuations in growth, as VWO represents emerging markets, and VEA represents a diversified fund of multiple global stock markets.

Sample Statistics

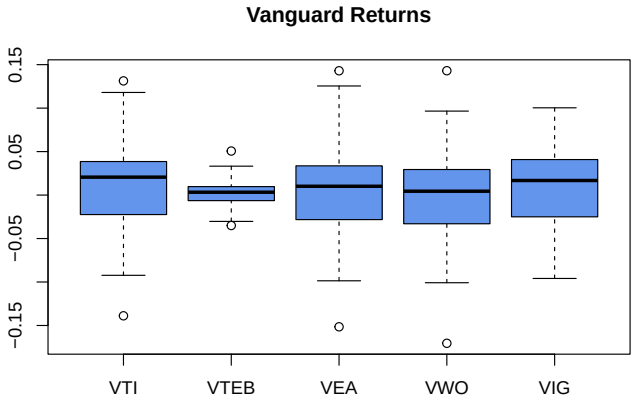
Here is a four panel plot:





From the four panel plots, we can see that none of the returns are normally distributed. While some are close, all depict some level of skew that offset the distributions. From the Q-Q plots, we can see that VTI and VTEB fall off the normal distribution at the negative extreme, with VTEB showing a large pivot from the normal distribution on the positive extreme. VEA has a relatively large positive skew, and VWO has the highest excess kurtosis out of the 5 stocks, indicating relatively fat tails. Lastly, VIG, VEA, and VTI all have significantly larger proportions of data in quartile 2 compared to quartile 3, indicating uneven distribution. In terms of time dependence, VTEB is the least likely to have linear time dependence, as it shows diminishing ACF. VTI, VWO and VIG show a slight pattern, without diminishing ACFs, indicating potential linear dependence. VEA is unclear, as it is unclear whether the ACF is diminishing or showing a pattern, indicating potential non-linear time dependence.

Group Boxplot



The boxplot further highlights that the distributions are likely not normal. We can see that VTI, VEA, and VIG especially show a large difference between Q2 and Q3, indicating uneven distributions. Out of the 5, VTI and VIG have the highest returns, but also have a higher distribution of negative returns. VTI, VEA,

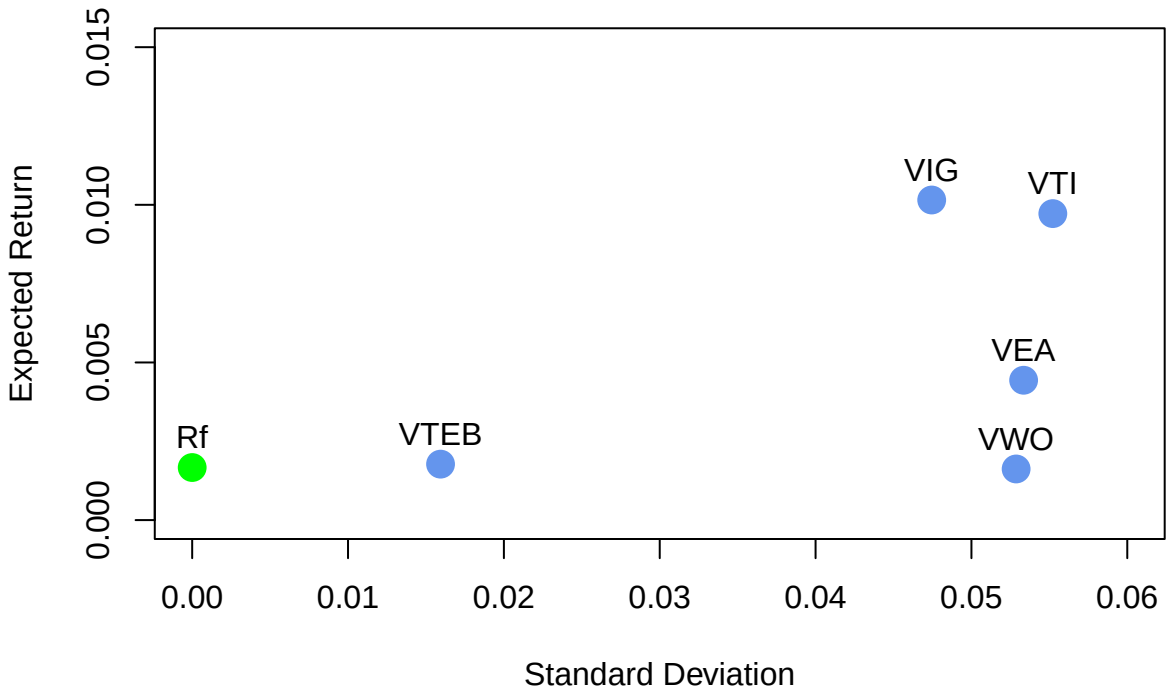
and VWO have the largest overall range, due to having outliers far out of the rest of the boxplot. VTEB has the smallest range, with relatively small outliers. However, the four panel plot clearly showcased a not normal distribution.

Univariate Descriptive Statistics

```
##                      VTI      VTEB      VEA      VWO      VIG
## Mean             0.00972  1.77e-03  0.00444  0.00162  0.01015
## Std Dev          0.05522  1.59e-02  0.05335  0.05287  0.04745
## Variance         0.00305  2.54e-04  0.00285  0.00279  0.00225
## Skewness        -0.42700  3.87e-05 -0.20135 -0.16008 -0.32404
## Excess Kurtosis  0.01733  8.48e-01  0.66158  1.09534 -0.38546
## 1% Quantile     -0.11139 -3.22e-02 -0.12030 -0.12938 -0.09093
## 5% Quantile     -0.09134 -2.74e-02 -0.08626 -0.06704 -0.08190
```

Out of the 5 funds, VIG has the highest expected return. VTI has a slightly lower expected return, but also comes with more risk. VEA and VWO both perform poorly - they have similar risk levels as VTI, but very low expected return. VTEB is the safest fund of the 5, having very low risk but also lower expected return. Most of the funds show negative skewness, except VTEB, which shows a very slight positive skewness. VWO and VTEB have the highest excess kurtosis, showcasing fat tails, while the others have almost 0 or negative excess kurtosis.

Risk-return graph



The analysis of the sample statistics above can be visualized through this risk-return graph. We can see that VIG, VTI, VEA, AND VWO all come with similar levels of risk, but VIG and VTI have higher expected returns. VTEB shows low risk and low returns. This graph generally follows the risk-return tradeoff - higher returns come with higher risk. However, VEA and VWO do not have high returns but do have high risk, indicating poor performance.

Overall, VTI followed by VIG seem to be the most normally distributed, and VTEB is the least, showing deviations from the normal distribution on both extremes.

Confidence Intervals

```
##           Mu    seMu  lowerMu upperMu widthMu     SD    seSD lowerSD
## VTI  0.00972 0.00713 -0.00454 0.02398 0.02852 0.0552 0.00504  0.0451
## VTEB 0.00177 0.00206 -0.00234 0.00589 0.00823 0.0159 0.00145  0.0130
## VEA  0.00444 0.00689 -0.00934 0.01821 0.02755 0.0533 0.00487  0.0436
## VWO  0.00162 0.00683 -0.01203 0.01527 0.02730 0.0529 0.00483  0.0432
## VIG  0.01015 0.00613 -0.00210 0.02240 0.02450 0.0475 0.00433  0.0388
##      upperSD widthSD
## VTI   0.0653 0.02016
## VTEB  0.0188 0.00582
## VEA   0.0631 0.01948
## VWO   0.0625 0.01930
## VIG   0.0561 0.01733
```

```
##  VTI VTEB  VEA  VWO  VIG
## 3.29 3.29 3.29 3.29 3.29
```

Overall, the means are not estimated very precisely. VTI and VIG are estimated the best out of the means, but the standard error is only slightly smaller than the mean, indicating a large error window. VTEB, VEA, and VWO all have standard errors that are larger than the mean, indicating poor estimates. Standard deviation is estimated a lot better than the mean - with all ETFs having standard errors lower than their standard deviation by a decimal point. Overall, standard deviation is estimated better than the mean.

Sharpe Ratio

```
##         Sharpe    SE LCL (0.95) UCL (0.95)
## VTI   0.145795 0.141    -0.1394      0.414
## VTEB  0.006570 0.132    -0.2512      0.264
## VEA   0.051830 0.127    -0.1965      0.302
## VWO  -0.000916 0.133    -0.2659      0.256
## VIG   0.178776 0.137    -0.0971      0.440
```

Here, VIG has the highest Sharpe ratio (slope), and VWO has the least, having a negative Sharpe ratio. The standard error for VTI and VIG are smaller than the Sharpe ratio, while for the others, they are higher. This indicates poor estimates for VTEB, VEA and VWO. It could be possible that assets with higher Sharpe ratios are being predicted better here.

Square Root of Time

```
##      muhat.vals annual.muhat annual.sigmahat annual.sr
## VTI     0.00972       0.1167          0.1913   0.50526
## VTEB    0.00177       0.0213          0.0552   0.02348
## VEA     0.00444       0.0532          0.1848   0.17976
## VWO     0.00162       0.0195          0.1831  -0.00295
## VIG     0.01015       0.1218          0.1644   0.61954
```

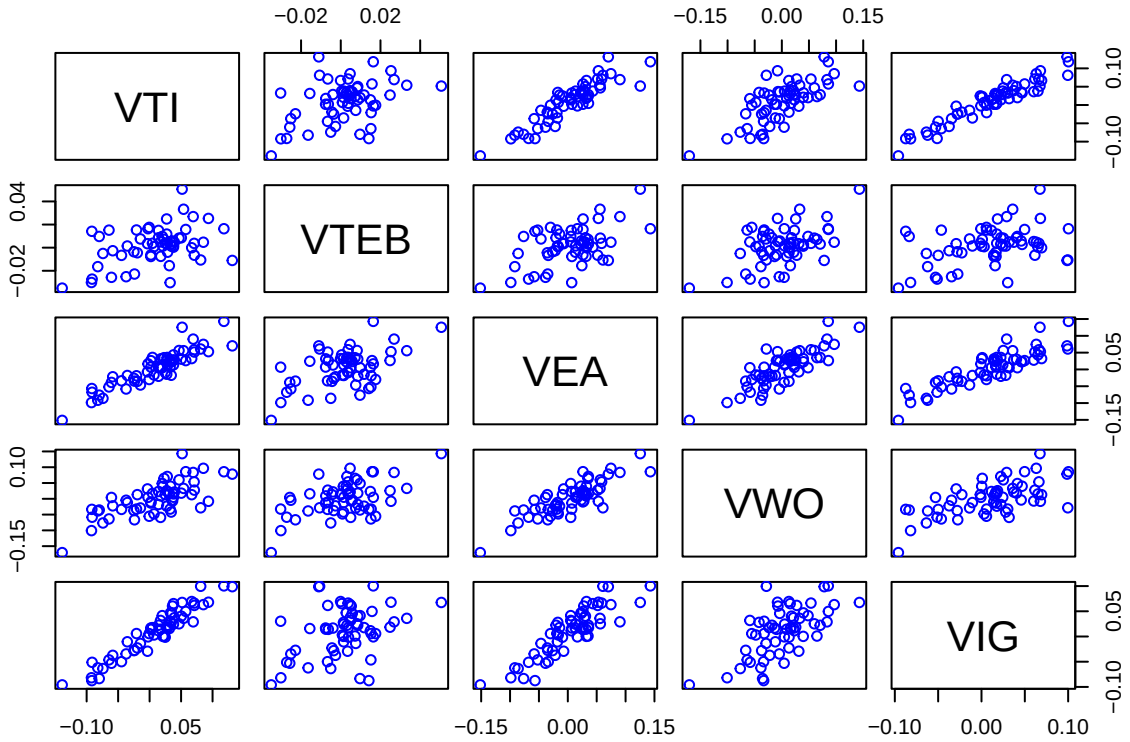
Yes, the ranking of values of the Sharpe ratio have stayed the same after annualization. These values seem reasonable.

```
(1 + mean(muhat.vals))^5
```

```
## [1] 1.03
```

After 5 years of annual returns on $1 at the average annual return, we would have $1.03.

Scatterplots



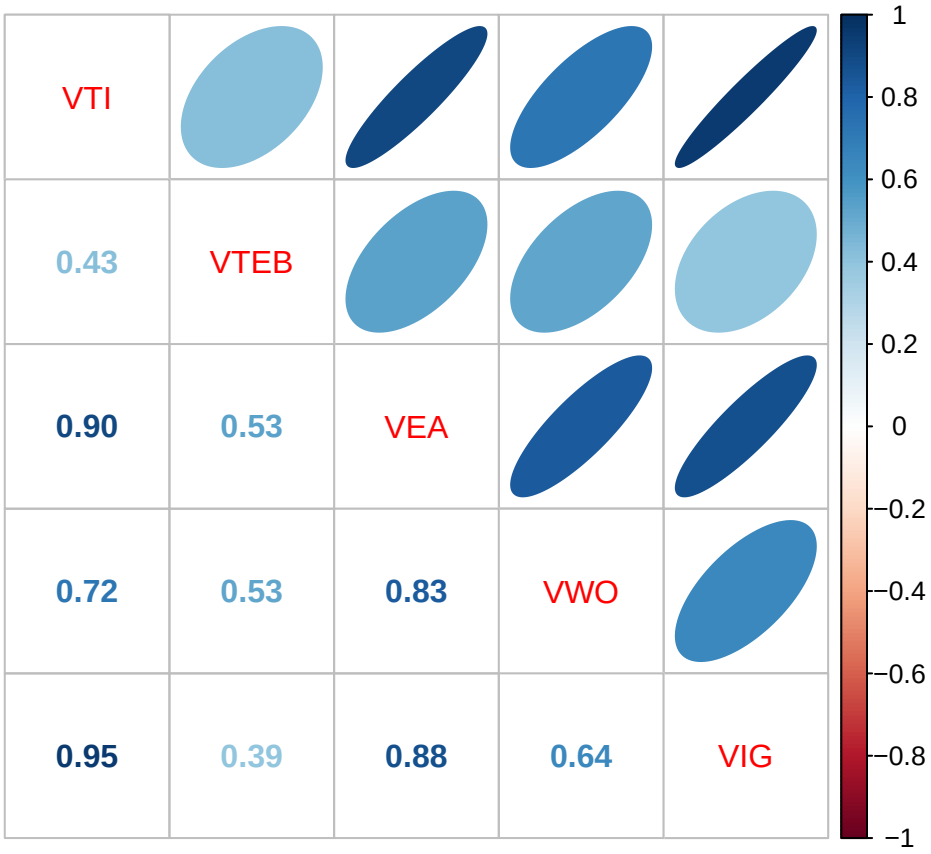
The scatterplots show distribution clouds of data that are positively sloped, with some forming a line. However, those that are more spread out still seem to follow a positive distribution.

Covariance Matrix

```
##            VTI      VTEB       VEA       VWO       VIG
## VTI   0.003049 0.000375 0.002664 0.002104 0.002497
## VTEB 0.000375 0.000254 0.000453 0.000443 0.000295
## VEA   0.002664 0.000453 0.002846 0.002351 0.002221
## VWO   0.002104 0.000443 0.002351 0.002795 0.001610
## VIG   0.002497 0.000295 0.002221 0.001610 0.002252
```

All values in the covariance matrix are positive, indicating that all the ETFs move in the same direction.

Correlation Matrix



The correlation matrix shows that all the assets at the least showcase some positive correlation with eachother, with most having significant positive correlation. VTI and VIG display the strongest correlation, having a data point distribution cloud that forms the shape of a line. VEA and VWO also show high positive correlation with VTI, VIG, and with eachother, likely since all 4 of these ETFs involve riskier assets that are more susceptible to market forces, while VTEB only consists of government bonds. For this reason, diversification of a portfolio with these 5 ETFs is not recommended, as they move in the same direction. For diversification, it is ideal to use funds that are negatively correlated with eachother.

Value-at-Risk Calculations

```
##      VaR.01 VaR.05
## VTI  -11874  -8111
## VTEB  -3530  -2444
## VEA  -11967  -8332
## VWO  -12137  -8534
## VIG  -10023  -6790
```

Out of the 5 ETFs, VWO has the highest Value-at-Risk, and VTEB has the least. This is likely due to VWO's position as a fund with high risk but low return, and VTEB's position as a government bond fund with significantly lower risk and return than the other 4.

Bootstrapping

```
##    ETF X1..VaR X5..VaR
## 1  VTI  -11874   -8111
## 2 VTEB   -3530   -2444
```

```
## 3  VEA  -11967   -8332
## 4  VWO  -12137   -8534
## 5  VIG  -10023   -6790
```

```
##    ETF X5..VaR   SE X95..Confidence.Interval
## 1  VTI   -8111 1264          (-10791, -5798)
## 2 VTEB   -2444  357           (-3192, -1795)
## 3  VEA   -8332 1216           (-1086, -6067)
## 4  VWO   -8534 1226          (-11182, -6271)
## 5  VIG   -6790  998           (-8957, -4891)
```

Overall, the standard error for the Value-at-Risk calculations is low. The 5% VaR estimate for each ETF falls within the confidence interval. At the 0.01 quantile, the ETFs have approximately 10% standard error. This increases for the 0.05 quantile, but the standard error is still significantly lower than the Value-at-Risk estimate, indicating a relatively good estimator.

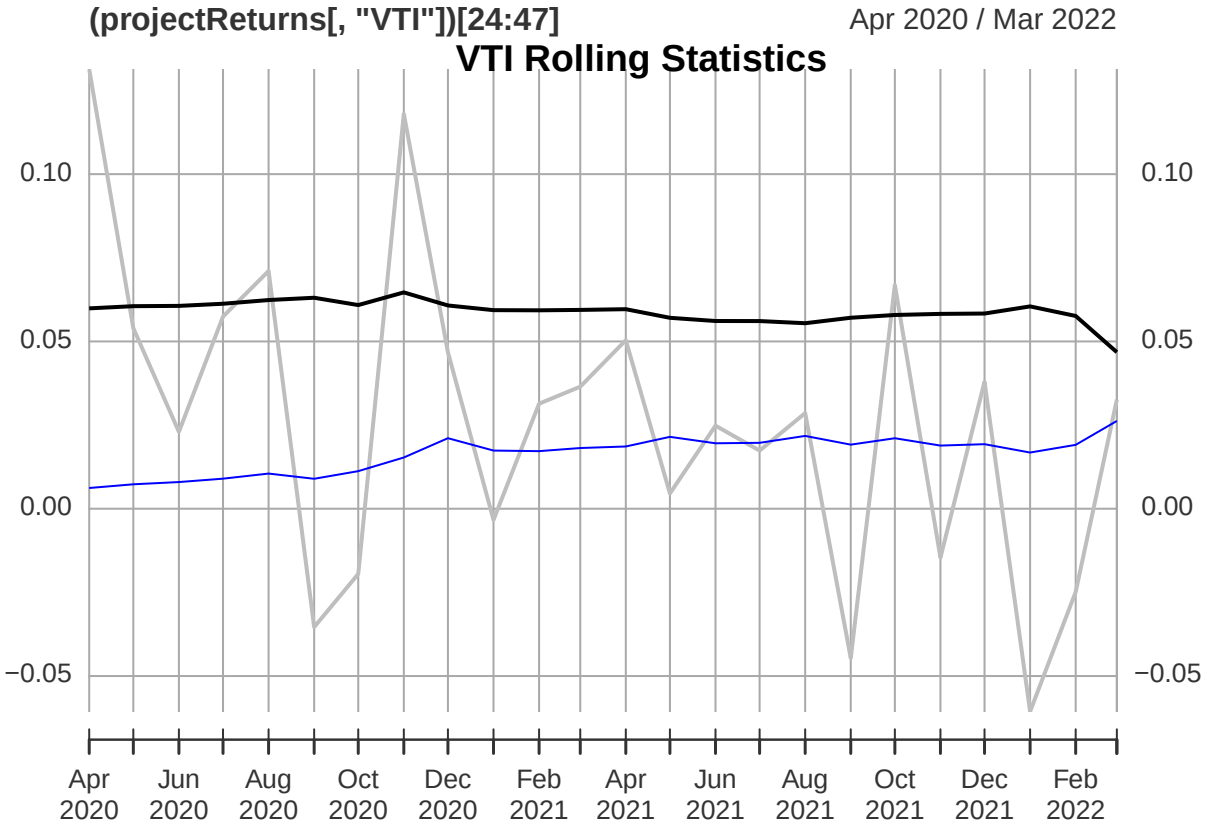
Historical VaR

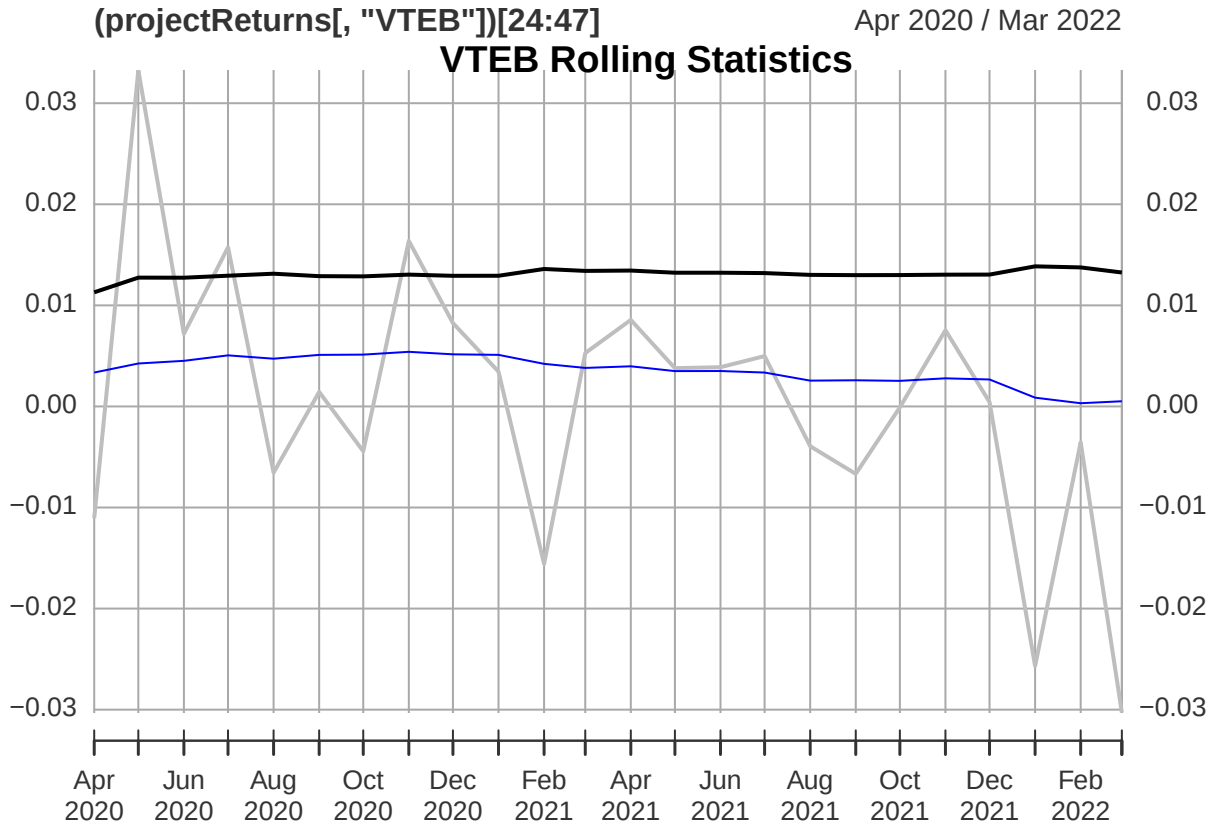
```
##    VTI.VaR VTEB.VaR VEA.VaR VWO.VaR VIG.VaR
## 1%  -11139    -3223  -12030  -12938   -9093
## 5%   -9134    -2738   -8626   -6704   -8190
```

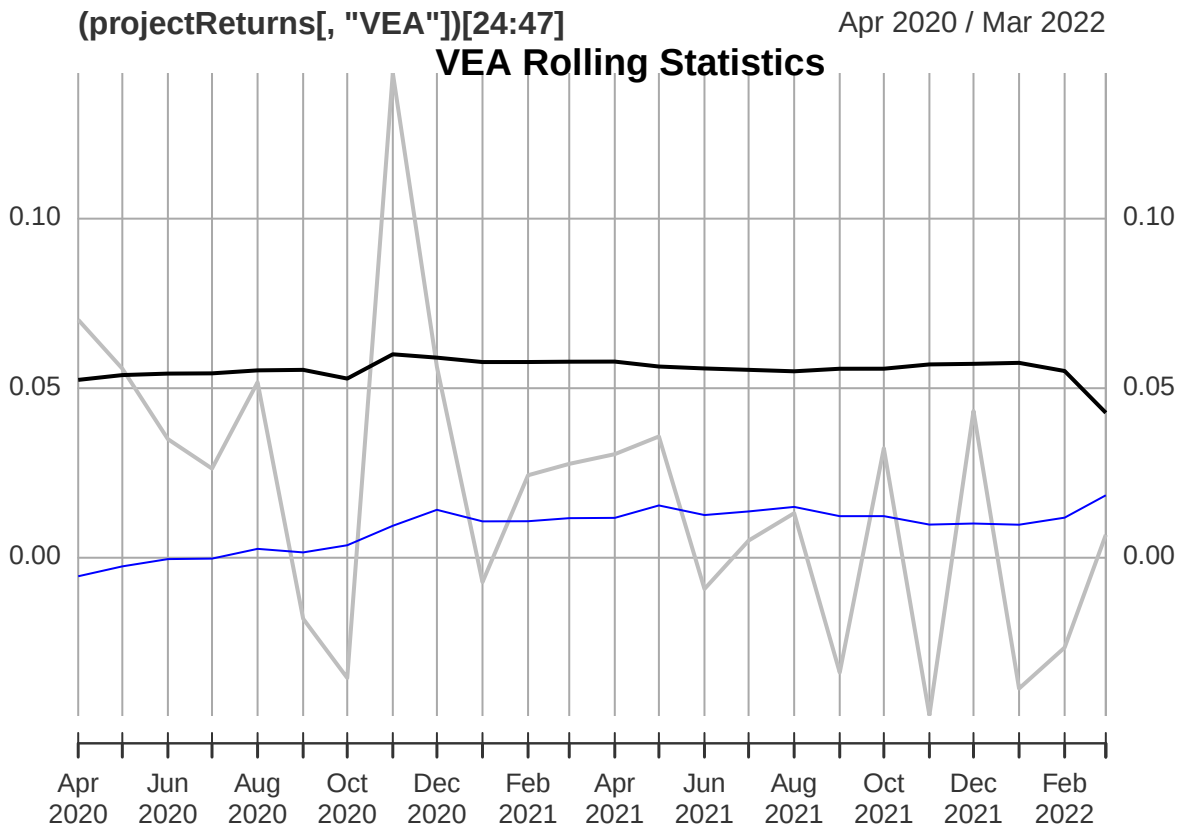
When compared to the VaR based on the normal distribution, VTEB, VTI and VIG have lower historical VaR for the 0.01 quantile, but higher VaR for the 0.05 quantile. For VEA, both the 0.01 and 0.05 quantiles for historical VaR show higher value at risk than the normal distribution. VWO has a higher VaR at the 0.01 quantile, but lower at the 0.05 quantile.

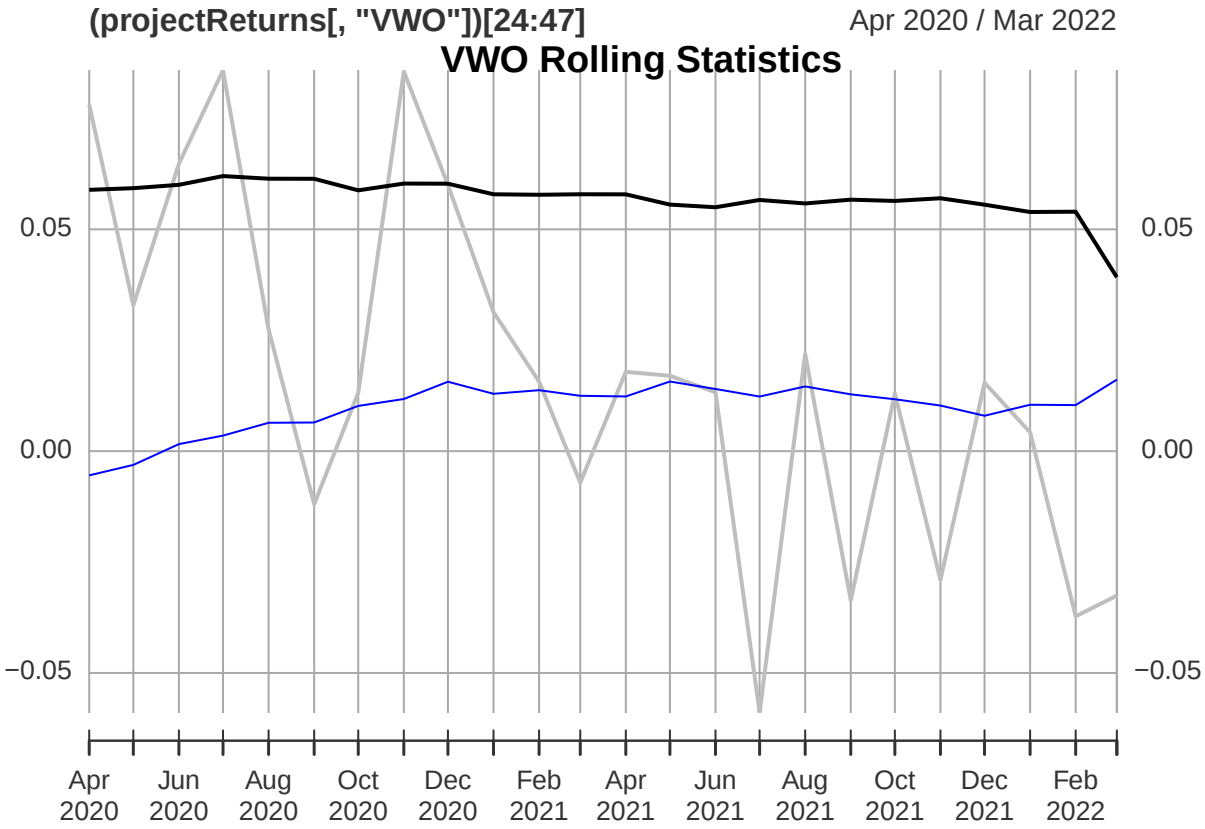
Rolling Sample Statistics

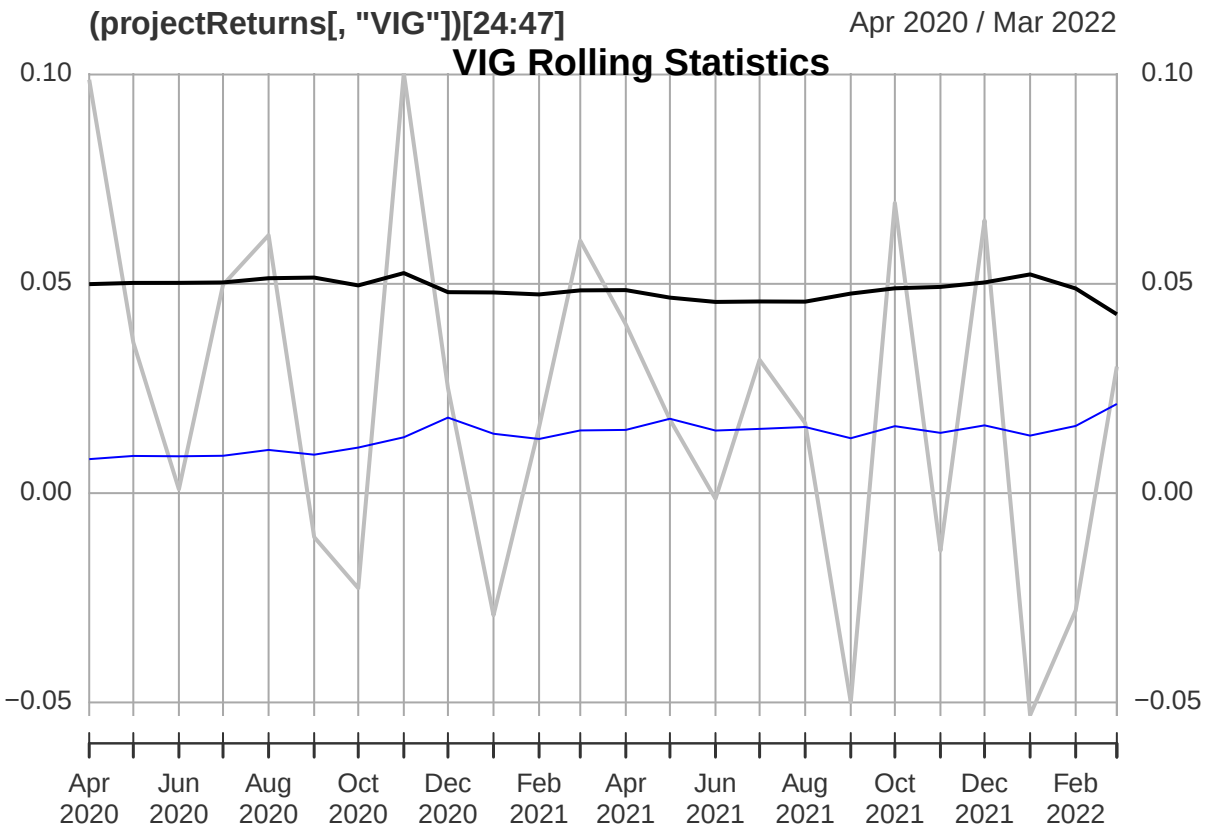
In this section, grey indicates the returns, black indicates risk, and blue indicates expected return.




(projectReturns[, "VTEB"])[24:47]
Apr 2020 / Mar 2022
VTEB Rolling Statistics
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
Apr 2020
Jun 2020
Aug 2020
Oct 2020
Dec 2020
Feb 2021
Apr 2021
Jun 2021
Aug 2021
Oct 2021
Dec 2021
Feb 2022


(projectReturns[, "VEA"])[24:47]
Apr 2020 / Mar 2022
VEA Rolling Statistics
0.10
0.05
0.00
Apr 2020
Jun 2020
Aug 2020
Oct 2020
Dec 2020
Feb 2021
Apr 2021
Jun 2021
Aug 2021
Oct 2021
Dec 2021
Feb 2022





While the rolling means and standard deviations show only slight changes over the 24 months, they still do

not seem constant. Excluding VTEB, all graphs show a slow increase in mean and an even slower decrease in standard deviation through December 2020, after which they stayed constant for a period of time. However, at the end of the graph, we can see a shift starting in January of 2022. When considering the world events discussed at the start of this paper, we can see that while the mean and standard deviation don't show drastic changes, they have some dependence on time. While VTEB shows relatively constant mean and risk up till December 2021, it also experiences the same shift seen in other graphs in December 2021/January 2022. This could indicate that it is just less sensitive to economic changes, since it consists of government bonds. Overall, these ETFs do not seem to be covariance stationary as they do not show constant expected return and risk over time.

Portfolio Theory

Equally Weighted Portfolio

```
## Call:
## getPortfolio(er = muhat.mat, cov.mat = cov.mat, weights = rep(1/5,
##     5))
##
## Portfolio expected return:     0.00554
## Portfolio standard deviation:  0.0406
## Portfolio weights:
## [1] 0.2 0.2 0.2 0.2 0.2
```

Annualizing:

```
##      annual_ret annual_sd annual_sr
## [1,]     0.0665     0.141     0.461
```

```
##                             Mean     SD Sharpe Ratio
## VTI                       0.1167 0.1913      0.50526
## VTEB                      0.0213 0.0552      0.02348
## VEA                       0.0532 0.1848      0.17976
## VWO                       0.0195 0.1831     -0.00295
## VIG                       0.1218 0.1644      0.61954
## Annual Equally Weighted 0.0665 0.1406      0.46081
```

From the table, we can see that the annual expected return, standard deviation and Sharpe ratio are both within the range of the returns and standard deviations of the assets.

VaR:

```
##      VaR.01 VaR.05
## [1,]  -8891  -6124
```

The equally weighted portfolio has a lower VaR than the assets on their own. This means that the equally weighted portfolio is able to reduce the risk to some degree.

Global Minimum Variance Portfolio

```
## Call:
## globalMin.portfolio(er = muhat.vals, cov.mat = cov.mat)
##
## Portfolio expected return:     0.00279
## Portfolio standard deviation:  0.0145
## Portfolio weights:
##     VTI    VTEB     VEA     VWO     VIG
## -0.1084  1.0249 -0.2488  0.0285  0.3038
```

```
##      Mean     SD
```

```
## 1 0.00279 0.0145
```

In the global minimum variance portfolio, VTI and VEA have negative weights, indicating that the portfolio was constructed by shorting these two funds. Since these are ETFs, this is replicable.

Annualizing:

```
##      annual_ret annual_sd annual_sr
## [1,]     0.0335    0.0502     0.633
```

```
##          Mean     SD Sharpe Ratio
## VTI    0.1167 0.1913      0.50526
## VTEB   0.0213 0.0552      0.02348
## VEA    0.0532 0.1848      0.17976
## VWO    0.0195 0.1831     -0.00295
## VIG    0.1218 0.1644      0.61954
## Annual 0.0335 0.0502      0.63329
```

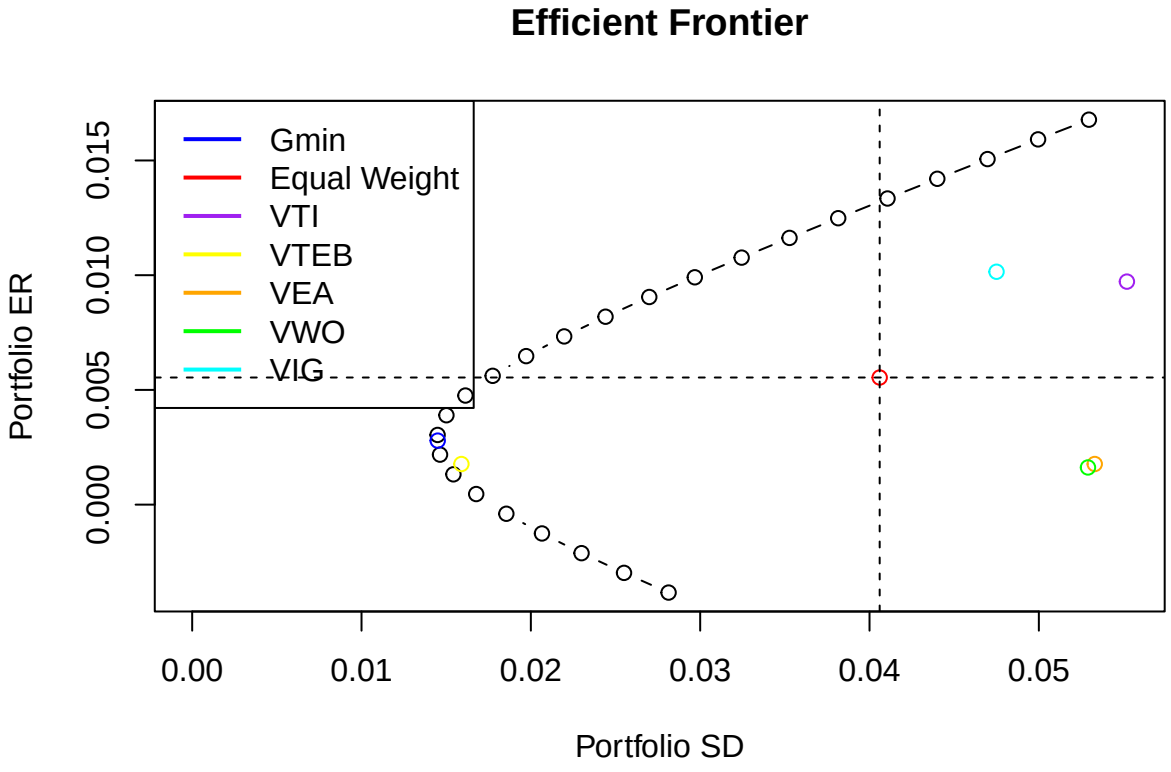
From the table, we can see that the annualizaed returns are within the range of the annualized expected returns of each asset. However, the standard deviation is lower than all of the asset standard deviations. This is characteristic of a global minimum variance portfolio, indicating that it has the least risky combination of the ETFs. The Sharpe ratio is also higher than those of the assets, indicating higher value.

VaR:

```
##      VaR.01 VaR.05
## [1,]  -3094  -2106
```

Here, we can see that the VaR of the global minimum variance portfolio is lower than that of the other assets There is a clear impact on risk here, as we can see a significantly lower standard deviation and value at risk, while still having a high Sharpe ratio.

Efficient Frontier



The efficient portfolio with the same expected return as the equally weighted portfolio would be port 9, which has an expected return of 0.0053, and a standard deviation of 0.0171. This portfolio has a standard deviation 0.0235 lower than that of the equal weight portfolio. The efficient portfolio with the same volatility as the equally weighted portfolio would be port 19, which has an expected return of 0.0131 and a standard deviation of 0.0401. This is 0.0077 higher than the expected return of the equally weighted portfolio.

Tangency Portfolio

```
## Call:
## tangency.portfolio(er = muhat.mat, cov.mat = cov.mat, risk.free = 0.00167)
##
## Portfolio expected return:     0.0168
## Portfolio standard deviation:  0.0532
## Portfolio weights:
## [1]  0.2968  0.3926 -1.8389  0.0474  2.1021
```

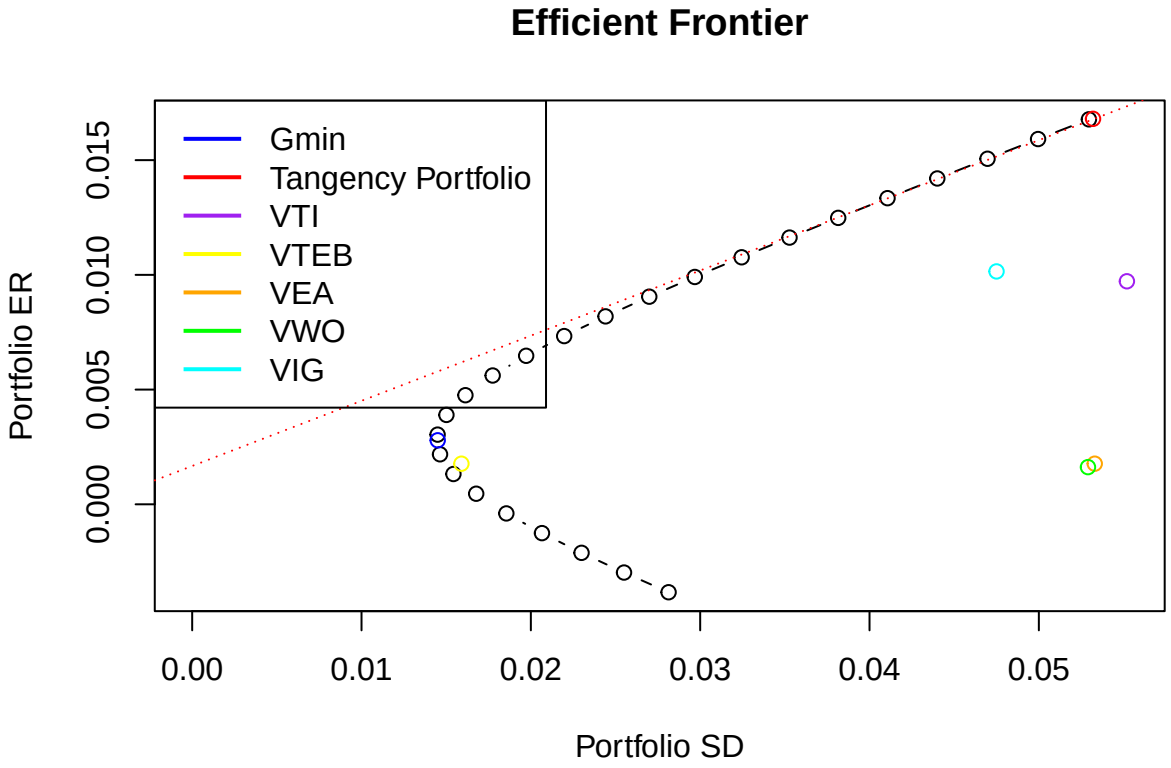
On the tangency portfolio, we have one asset, VEA, with a negative weight, indicating it is being shorted. The expected return of this portfolio is 0.0168, and the standard deviation is 0.0532.

Sharpe Ratio:

```
(0.0168-0.00167) / 0.0532
```

```
## [1] 0.284
```

```
## Call:
## efficient.frontier(er = muhat.mat, cov.mat = cov.mat, nport = 25,
##     alpha.min = -0.9, alpha.max = 1.9)
##
## Frontier portfolios' expected returns and standard deviations
##     port 1  port 2  port 3  port 4  port 5 port 6 port 7 port 8
## ER -0.0038 -0.0030 -0.0021 -0.0013 -0.0004 0.0005 0.0013 0.0022
## SD  0.0281  0.0255  0.0230  0.0207  0.0186 0.0168 0.0154 0.0146
##    port 9 port 10 port 11 port 12 port 13 port 14 port 15 port 16
## ER 0.0030  0.0039  0.0048  0.0056  0.0065  0.0073  0.0082   0.009
## SD 0.0145  0.0150  0.0161  0.0177  0.0197  0.0220  0.0244   0.027
##    port 17 port 18 port 19 port 20 port 21 port 22 port 23 port 24
## ER  0.0099  0.0108  0.0116  0.0125  0.0133  0.0142  0.0151  0.0159
## SD  0.0297  0.0324  0.0353  0.0382  0.0411  0.0440  0.0470  0.0500
##    port 25
## ER  0.0168
## SD  0.0530
```



Here, the tangency portfolio is marked with the red line and red dot. It has a standard deviation of 0.0532, and an expected return of 0.0168. The Sharpe ratio is 0.284.

Annualizing:

```
##      annual_ret annual_sd annual_sr
## [1,]     0.0335    0.0502     0.633
```

When annualized, the Sharpe ratio increases from 0.284 to 0.633. This shows that the tangency portfolio is beneficial especially in long-term investments.

Global Minimum Variance - No Short Sales

```
## Call:
## globalMin.portfolio(er = muhat.mat, cov.mat = cov.mat, shorts = FALSE)
##
## Portfolio expected return:     0.00177
## Portfolio standard deviation:  0.0159
## Portfolio weights:
## [1] 0 1 0 0 0
```

```
##      Mean     SD
## 1 0.00177 0.0159
```

In the global minimum variance portfolio with no short sales, there is only one asset involved within the porfolio, VTEB. Since VTEB has a significantly lower standard deviation than the other 4 ETFs, it makes sense that the global minimum variance portfolio would only utilize this when there are short sales constraints.

Annualizing:

```
##      monthly_sr annual_ret annual_sd annual_sr
## [1,]    0.00629     0.0212    0.0551     0.355
```

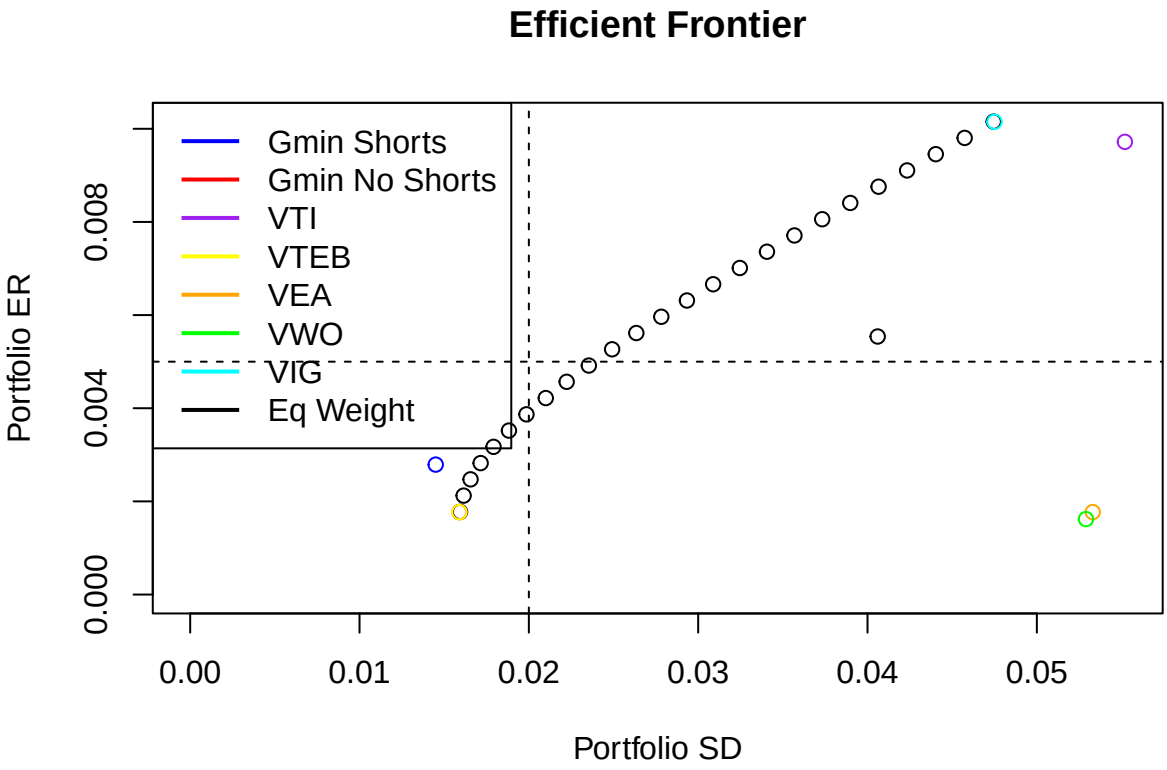
Compared to the global minimum variance portfolio which allowed shorts, this portfolio has a lower expected return, but also a higher standard deviation. This indicates the risk reduction that comes from diversification through short-selling. It also has a very small Sharpe ratio, indicating that this portfolio is not efficient in terms of balancing risk and returns.

VaR:

```
##      VaR.01 VaR.05
## [1,]  -3522  -2438
```

From the VaR, we can see that the VaR is still lower than that of all the assets on their own, but it has a higher VaR than the global minimum variance portfolio with short sales.

Efficient Frontier - No Short Sales



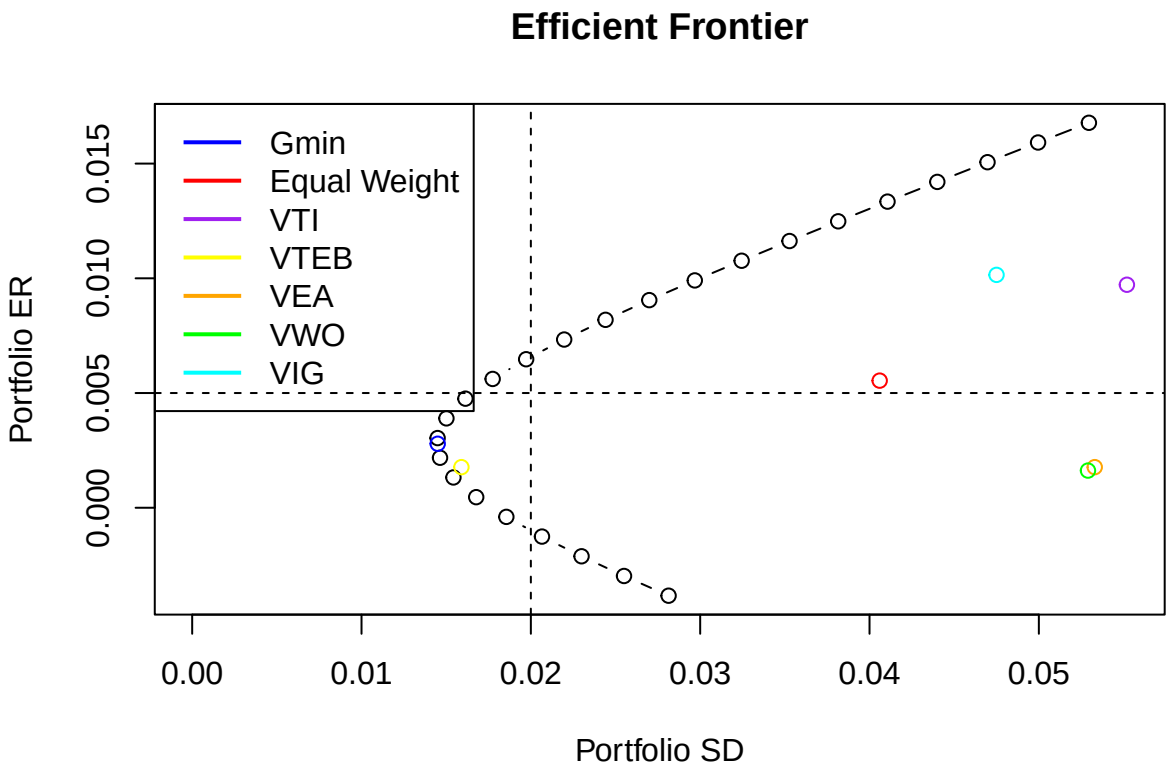
From the graph, we can see that the global minimum portfolio without shorts overlaps with the point for VTEB, indicating that 100% of the weight in the portfolio is in VTEB. It has a higher SD, and lower expected return than the global minimum portfolio with shorts.

Target Volatility of 0.02

Let's look at the short sales graph:

```
## Call:
## efficient.frontier(er = muhat.mat, cov.mat = cov.mat, nport = 25,
##     alpha.min = -0.9, alpha.max = 1.9)
##
## Frontier portfolios' expected returns and standard deviations
##     port 1  port 2  port 3  port 4  port 5 port 6 port 7 port 8
## ER -0.0038 -0.0030 -0.0021 -0.0013 -0.0004 0.0005 0.0013 0.0022
## SD  0.0281  0.0255  0.0230  0.0207  0.0186 0.0168 0.0154 0.0146
##    port 9 port 10 port 11 port 12 port 13 port 14 port 15 port 16
## ER 0.0030  0.0039  0.0048  0.0056  0.0065  0.0073  0.0082   0.009
```

```
## SD 0.0145  0.0150  0.0161  0.0177  0.0197  0.0220  0.0244   0.027
##    port 17 port 18 port 19 port 20 port 21 port 22 port 23 port 24
## ER  0.0099  0.0108  0.0116  0.0125  0.0133  0.0142  0.0151  0.0159
## SD  0.0297  0.0324  0.0353  0.0382  0.0411  0.0440  0.0470  0.0500
##    port 25
## ER  0.0168
## SD  0.0530
```



On the graph that allows short sales, portfolio 13 has a volatility of 0.02, and an expected return of 0.0065. On the graph that does not allow short sales, portfolio 7 has a volatility of 0.0199 and an expected return of 0.0039. Choosing the portfolio that does not allow short sales results in a loss of expected return of 0.0026.

Target Expected Return of 0.005

On the graph that allows short sales, portfolio 11 has an expected return of 0.0048, and a volatility of 0.0161. On the graph that does not allow short sales, portfolio 10 has an expected return of 0.0049, and a volatility of 0.0235. Selecting a portfolio that does not allow short sales results in an increase in volatlity of 0.0074.

Tangency Portfolio - No Short Sales

```
## Call:
## tangency.portfolio(er = muhat.mat, cov.mat = cov.mat, risk.free = 0.00167,
##     shorts = FALSE)
##
## Portfolio expected return:     0.0102
## Portfolio standard deviation:  0.0475
## Portfolio weights:
## [1] 0 0 0 0 1
```

```
##      monthly_ret monthly_sd monthly_sr
## [1,]      0.0102     0.0475       0.18
```

Annualizing:

```
##      annual_ret annual_sd annual_sr
## [1,]     0.0335    0.0502     0.633
```

On a tangency portfolio with no short sales, 100% of the weight goes to the VIG ETF. It has an expected return of 0.0102, a standard deviation of 0.0475, and a Sharpe ratio of 0.18. When annualized, the tangency portfolio has a Sharpe ratio of 0.633, indicating high long-term value. When compared to the tangency portfolio that allows short sales, the expected return, standard deviation and Sharpe ratio are all significantly lower. Overall, we can see the benefit of using short-sales to diversify a portfolio and obtain a better level of risk and reward.

Risk Budgeting

```
##      Dollar Weight    Vol     MCR      CR    PCR
## VTI   2e+04    0.2 0.0552 0.05265 0.01053 0.2593
## VTEB  2e+04    0.2 0.0159 0.00896 0.00179 0.0442
## VEA   2e+04    0.2 0.0533 0.05189 0.01038 0.2556
## VWO   2e+04    0.2 0.0529 0.04582 0.00916 0.2257
## VIG   2e+04    0.2 0.0475 0.04371 0.00874 0.2153
## PORT  1e+05    1.0     NA      NA 0.04061 1.0000
```

Excluding VTEB, there is an approximately equal distribution of risk across all assets. This aligns with the volatility, as all assets except VTEB have similar volatility. VTI and VEA both contribute 25-26% each to the overall risk of the portfolio, while VWO and VIG contribute 21-22 percent of risk to the portfolio. VTEB only contributes to 4% of risk however, even though it is at an equal weight. This is due to VTEB consisting of government bonds. Overall, the risk is generally well balanced in this portfolio. However, VTI and VEA marginally contribute more to risk than the other assets.

Asset Allocation

Target Expected Return of 0.5% (No Shorts)

```
##                           V1
## Expected.Return     5.00e-03
## Standard.Deviation  2.39e-02
## VTI.Weight          0.00e+00
## VTEB.Weight         6.15e-01
## VEA.Weight          0.00e+00
## VWO.Weight          0.00e+00
## VIG.Weight          3.85e-01
## VaR.1.             -5.06e+03
## VaR.5.             -3.43e+03
```

Target Expected Return of 0.5% (Only Tangency Portfolio No Shorts)

```
## Call:
## tangency.portfolio(er = muhat.mat, cov.mat = cov.mat, risk.free = 0.00167,
##     shorts = FALSE)
##
## Portfolio expected return:     0.0102
## Portfolio standard deviation:  0.0475
## Portfolio weights:
## [1] 0 0 0 0 1
```

```
##      monthly_ret monthly_sd monthly_sr
## [1,]      0.0102     0.0475       0.18
```

```
##                                                  V1
## Expected.Return                               0.005
```

```
## Standard.Deviation                                  0.0185
## Tangency.Weight                                      0.39
## T.Bill.Weight                                        0.61
## Tangency.Portfolio..VIG.Weight 1 (0 in everything else)
## VaR.1.                                              -3814
## VaR.5.                                              -2550
```

Target Expected Return of 1%

```
## Call:
## efficient.portfolio(er = muhat.vals, cov.mat = cov.mat, target.return = 0.01,
##     shorts = FALSE)
##
## Portfolio expected return:     0.01
## Portfolio standard deviation:  0.0467
## Portfolio weights:
##    VTI   VTEB    VEA    VWO    VIG
## 0.0000 0.0183 0.0000 0.0000 0.9817
```

In this case, this works, since the expected return of 0.01 (1%) is lower than the expected return of each individual ETF. However, if any of the ETFs had an expected return higher than 1%, it would not work as when we have short sales, there cannot be any portfolios with a higher expected return than the highest expected return out of the 5 ETFs.